Area and Volume

Objective 1

Calculate the Area of a Figure

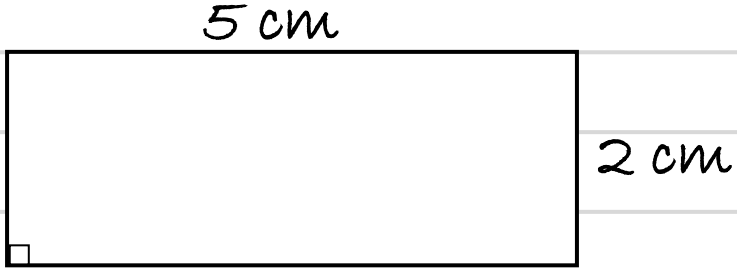
Recall that the perimeter of a figure is a one-dimensional length and is express as ft, cm, in., etc.

But area involves 2 dimensions. For rectangles we measure its length and width and then multiply the two together. Since we are multiplying these two lengths together, area is a two-dimensional quantity and is expressed as ft^2, cm^2, in^2, etc.

The formula for the area of a rectangle is:

$A = \text{length} \cdot \text{width}$ -or- $A = l \cdot w$

Example 1: Find the area of the given rectangle.

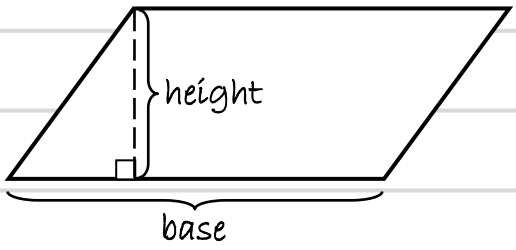


Note: Recall that $x \cdot x = x^2$.
Similarly, $cm \cdot cm = cm^2$.

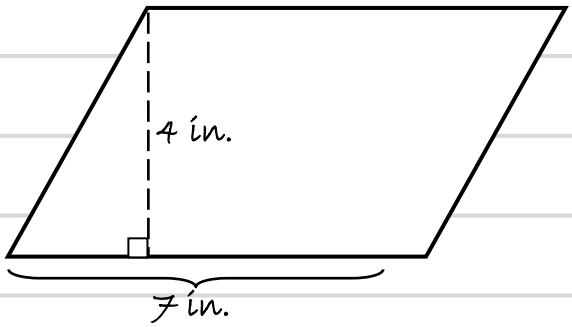
The term cm^2 is said "centimeters squared" or "square centimeters".

A **parallelogram** is a quadrilateral, where opposite sides are both parallel and have the same length.

The formula for the area of a parallelogram is:
$A = \text{base} \cdot \text{height}$ -or- $A = b \cdot h$



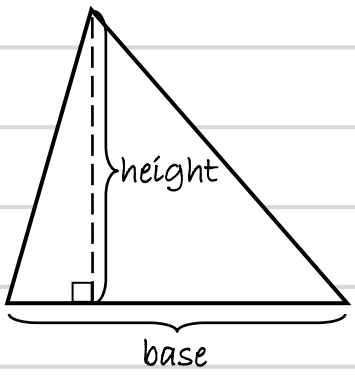
Example 2: Find the area of the given parallelogram.



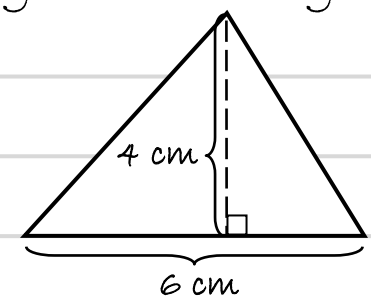
A **triangle** is a three sided polygon.

The formula for the area of a triangle is:

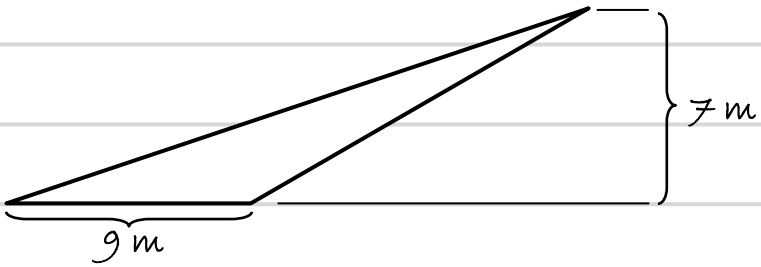
$A = (b \cdot h) \div 2$ -or- $A = \frac{1}{2}(b \cdot h)$



Example 3: Find the area of the given triangle.



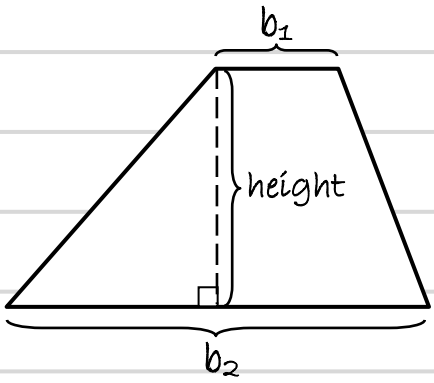
Example 4: Find the area of the given triangle.



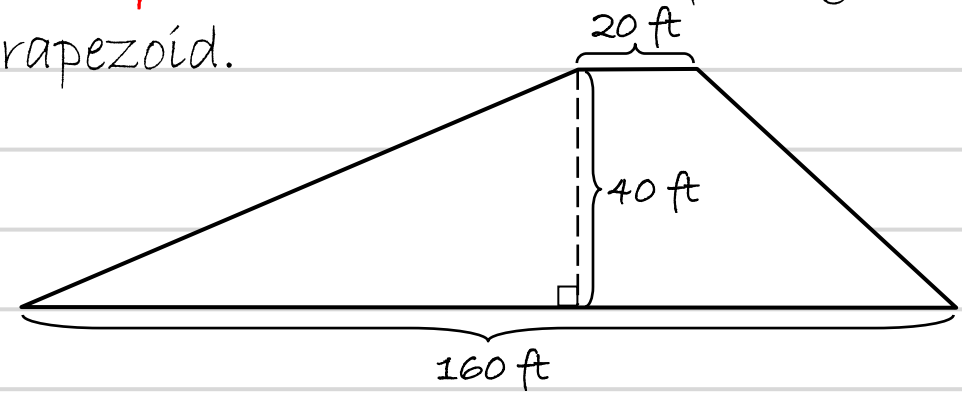
A **trapezoid** is a quadrilateral where exactly one pair of opposite sides are parallel. Unlike a parallelogram, opposite sides do not necessarily have the same length.

The formula for the area of a trapezoid is:

$A = [h \cdot (b_1 + b_2)] \div 2$ -or- $A = \frac{1}{2} \cdot h \cdot (b_1 + b_2)$



Example 5: Find the area of the given trapezoid.

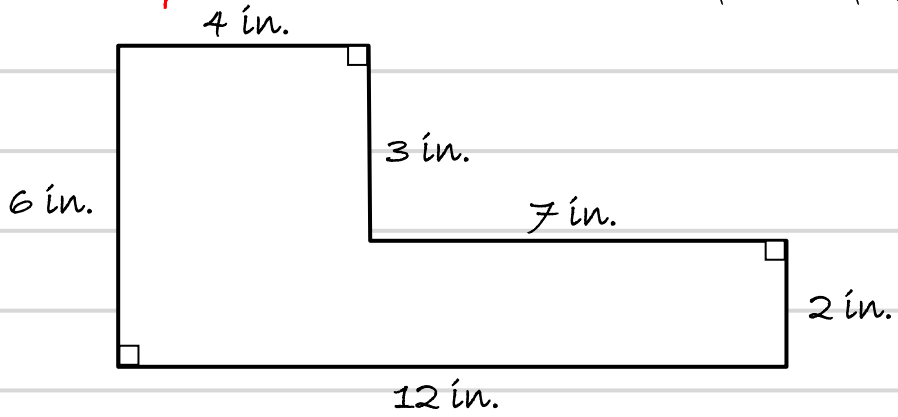


Objective 2

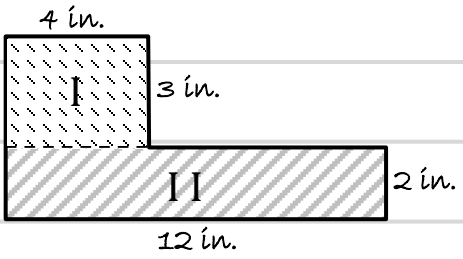
Calculate the Area of a Composite Figure

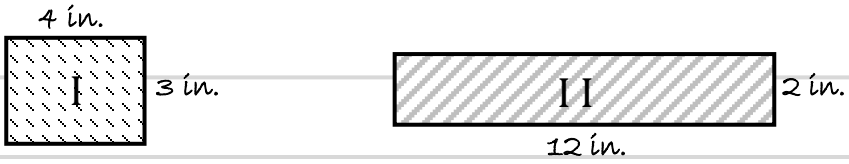
In many cases we need to partition our figure so that it consists of familiar shapes such as parallelograms, rectangles, trapezoids, or triangles. The total area is the sum of the individual areas.

Example 6: Find the area of the figure below.



Partition the figure into two rectangles. Notice that you only need the lengths related to the dimensions of each individual rectangle.



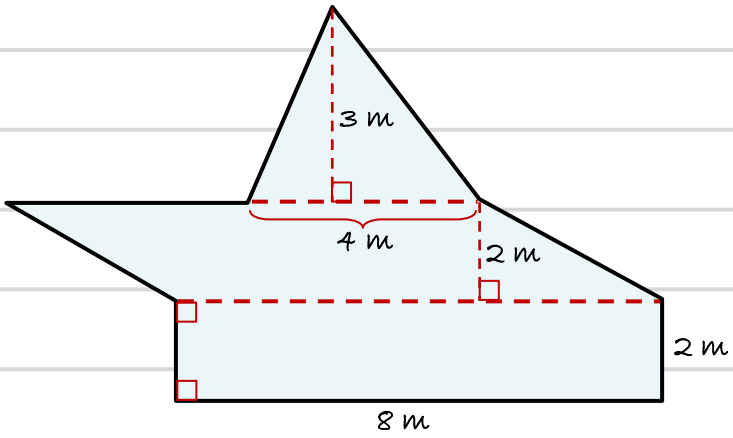


$A_1 = l \cdot w = 4\ in. \cdot 3\ in. = 12\ in^2$

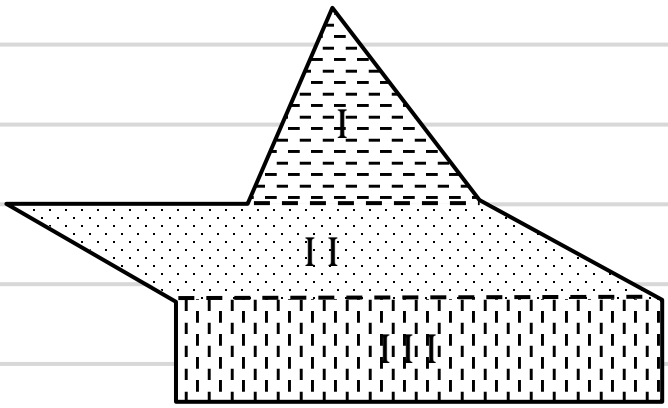
$A_2 = l \cdot w = 12\ in. \cdot 2\ in. = 24\ in^2$

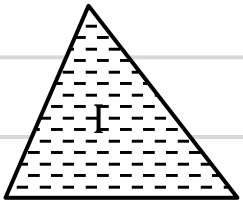
$A_{Total} = 12\ in^2 + 24\ in^2 = \mathbf{36\ in^2}$

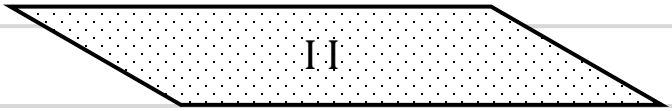
Example 7: Find the area of the figure below.

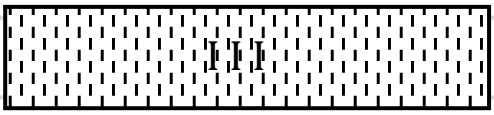


Partition the figure into three separate shapes; a triangle, parallelogram, and a rectangle. Find their individual areas and add them together to get the total area.









Objective 3

Calculate the Volume of a Rectangular Prism

Recall that the area of a figure is a two-dimensional quantity and is express as ft^2, cm^2, in^2, etc.

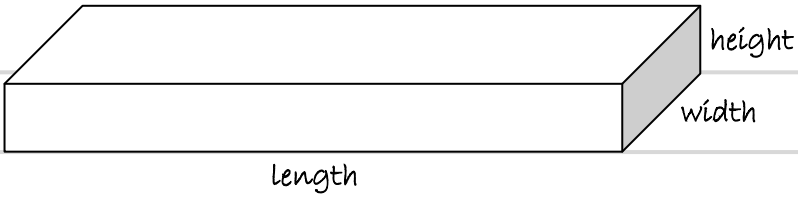
But volume involves 3 dimensions. For rectangles we measure its length, width, and height then find the product of these three lengths. Since we are multiplying these three lengths together, volume is a three-dimensional quantity and is expressed as ft^3, cm^3, in^3, etc.

The formula for the volume of a rectangular prism is:

$V = \text{length} \cdot \text{width} \cdot \text{height}$

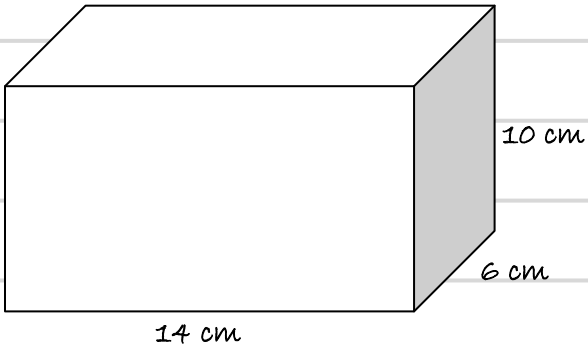
-or-

$A = l \cdot w \cdot h$

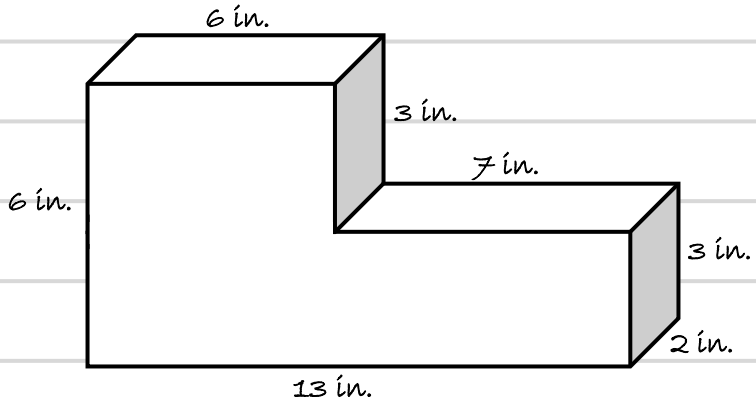


Note: Recall that $x \cdot x \cdot x = x^3$.
Similarly, $ft \cdot ft \cdot ft = ft^3$.

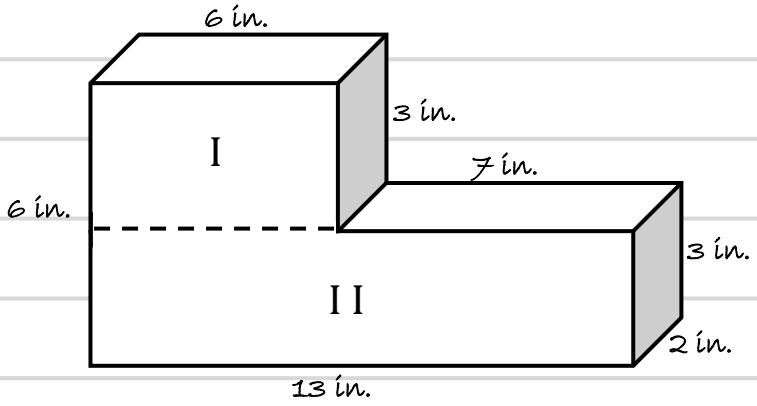
Example 8: Find the volume of the figure below.

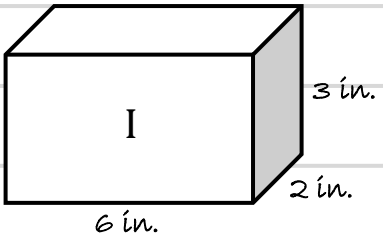


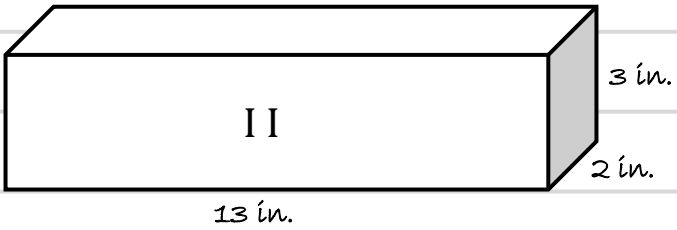
Example 9: Find the volume of the figure below.



To find the volume of this particular figure, we will partition the figure using a horizontal cut. We could also use a vertical cut if preferred.







$V_1 = l \cdot w \cdot h =$

$V_2 = l \cdot w \cdot h =$

$V_{Total} =$

Answer the following homework questions.

In Exercises 1 – 3, fill in the blank to make the statement true.

1) Perimeter has _____ dimension.

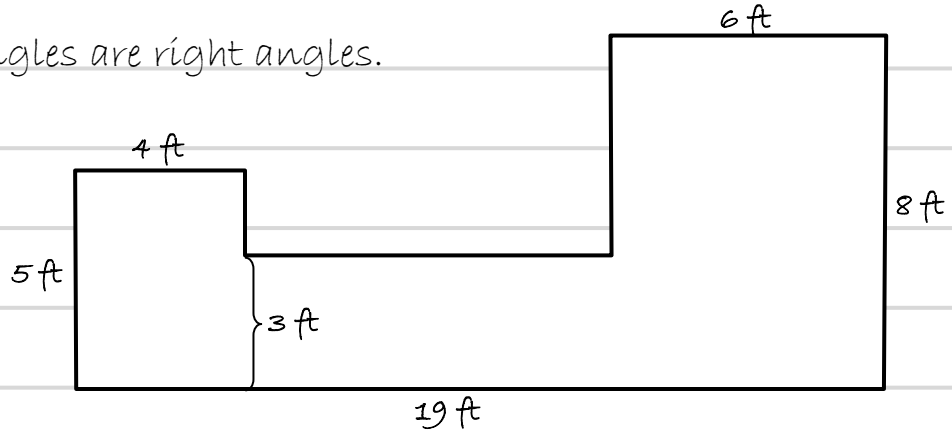
2) Area has _____ dimensions.

3) Volume has _____ dimensions.

Note: In Exercises 4 and 5, you will need to find missing side lengths.

4) Find the perimeter and area of the given figure.

All angles are right angles.



5) Find the volume of the given figure.